

# MATH 265: PRACTICE EXAM I

Name \_\_\_\_\_

Student # \_\_\_\_\_

	<b>Answer</b>	<b>Points awarded</b>
1. (10 points)	_____	_____
2. (10 points)	_____	_____
3. (10 points)	_____	_____
4. (10 points)	_____	_____
5. (10 points)	_____	_____
6. (10 points)	_____	_____
7. (10 points)	_____	_____
8. (10 points)	_____	_____
9. (10 points)	_____	_____
10. (10 points)	_____	_____
11. (10 points)	_____	_____
12. (10 points)	_____	_____
		<b>Total Points:</b> _____

1. If

$$\begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix} \begin{bmatrix} a & 1 \\ 0 & b \end{bmatrix} = \begin{bmatrix} 2 & 1 \\ 4 & 2 \end{bmatrix},$$

then

- A.  $a = 1, b = 1$
- B.  $a = 0, b = 1$
- C.  $a = 0, b = -1$
- D.  $a = 2, b = 0$
- E.  $a = 2, b = 1$

Correct ans is D.

2. Which of the following matrices is in row echelon form?

i.  $\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$

ii.  $\begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$

iii.  $\begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix}$

iv.  $\begin{bmatrix} 1 & 1 & 1 \\ 1 & 0 & 1 \end{bmatrix}$

- A. (i) and (ii)
- B. (i) and (iii)
- C. (ii) and (iii) only
- D. (ii) and (iv)
- E. (iv) only

Correct ans is A.

3. Determine all the values of  $a$  for which the system is inconsistent.

$$\begin{aligned} x + y - z &= 2 \\ -x - 2y + 2z &= 3 \\ x + y + (a^2 - 5)z &= a \end{aligned}$$

- A.  $a = 2$  only.

- B.  $a = -2$  only.
- C.  $a \neq 2$  or  $a \neq -2$ .
- D.  $a = 2$  or  $a = -2$ .
- E. None of the above.

Correct ans is B.

4. The matrix  $\begin{bmatrix} a & b \\ c & d \end{bmatrix}$  is nonsingular if and only if

- A.  $a \neq 0$  and  $b \neq 0$
- B.  $ad - bc \neq 0$
- C.  $a = b = c = d = 1$
- D.  $ad - bc = 0$
- E.  $a = b = c = d$

Correct ans is B.

5. If the system

$$\begin{aligned} x + 2y - 3z &= a \\ 2x + 3y + z &= b \\ 5x + 9y - 8z &= c \end{aligned}$$

is consistent, what can we conclude about  $a$ ,  $b$  and  $c$ ?

- A.  $3a - b + c = 0$ .
- B.  $a - 3b + c = 0$ .
- C.  $3a + b - c = 0$ .
- D.  $3a - b - c = 0$
- E.  $a + 3b - c = 0$

Correct ans is C.

6. Find all the values of  $a$  such that

$$A = \begin{bmatrix} 0 & 1 & a \\ 1 & 3 & 5 \\ a & 2 & a \end{bmatrix}$$

is singular.

- A.  $a = -1$  and  $a = 0$ .
- B.  $a = 0$  and  $a = -2$ .

C.  $a = 0$  and  $a = 2$ .

D.  $a = 2$ .

E.  $a = 0$  and  $a = 1$ .

Correct ans is C.

7. Evaluate the determinant

$$\begin{vmatrix} 1 & 1 & 0 & 2 \\ 0 & 1 & 0 & -1 \\ 3 & 2 & 4 & 0 \\ 0 & 3 & 0 & 2 \end{vmatrix}.$$

A. 6

B.  $-4$

C.  $-12$

D. 4

E. 20

Correct ans is E.

8. Which of the following matrices is adjoint to

$$\begin{bmatrix} 0 & 3 & 0 \\ -1 & 0 & 2 \\ 0 & 1 & -1 \end{bmatrix} ?$$

A.

$$\begin{bmatrix} 2 & -3 & -6 \\ 1 & 0 & 0 \\ 1 & 0 & 3 \end{bmatrix}.$$

B.

$$\begin{bmatrix} -2 & -3 & 6 \\ 1 & 0 & 0 \\ -1 & 0 & -3 \end{bmatrix}.$$

C.

$$\begin{bmatrix} -2 & 3 & 6 \\ -1 & 0 & 0 \\ -1 & 0 & 3 \end{bmatrix}.$$

D.

$$\begin{bmatrix} -2 & 1 & -1 \\ -3 & 0 & 0 \\ 6 & 0 & -3 \end{bmatrix}.$$

E.

$$\begin{bmatrix} -2 & -1 & -1 \\ 3 & 0 & 0 \\ 6 & 0 & -3 \end{bmatrix}.$$

Correct ans is C.

9. Let  $A$  be an  $n \times n$  invertible matrix. Which of the following statements are true?

- i.  $A^\top$  is singular.
- ii.  $A\mathbf{x} = \mathbf{0}$  has infinitely many solutions.
- iii.  $A$  is row equivalent to the identity matrix.
- iv.  $A\mathbf{x} = \mathbf{b}$  has a unique solution for every  $n \times 1$  matrix  $\mathbf{b}$ .
- v.  $\det(A) = 0$ .

- A. (i) and (iii).
- B. (ii) and (v).
- C. (iii) only.
- D. (iii) and (iv).
- E. (v) only.

Correct ans is D.

10. Which of the following statements about all  $5 \times 5$  matrices is **true**?

- A.  $\det(A - B) = \det A - \det B$
- B.  $\det(-A) = \det(A)$ .
- C.  $\det A^\top = \det(A)$ .
- D.  $AB = AC$  implies  $B = C$ .
- E.  $AB = 0$  implies  $A = 0$  or  $B = 0$ .

Correct ans is C.

11. Let  $V$  be the set of positive real numbers. Let the definition of vector addition be  $\mathbf{u} \oplus \mathbf{v} = \mathbf{uv}$  for every  $\mathbf{u}$  and  $\mathbf{v}$  in  $V$  and the definition of multiplication by a scalar be  $c \odot \mathbf{u} = \mathbf{u}^c$  for every real number  $c$  and every  $\mathbf{u}$  in  $V$ . Which statement below is **not** true?

- A.  $V$  is closed under vector addition.
- B.  $V$  is closed under scalar multiplication.
- C. The vector addition is commutative:  $\mathbf{u} \oplus \mathbf{v} = \mathbf{v} \oplus \mathbf{u}$  for every  $\mathbf{u}$  and  $\mathbf{v}$  in  $V$ .
- D. It is true that  $1 \odot \mathbf{u} = \mathbf{u}$  for every  $\mathbf{u}$  in  $V$ .
- E. It is true that  $0 \odot \mathbf{u} = \mathbf{0}$  for every  $\mathbf{u}$  in  $V$ .

Correct ans is E.

**12.** Which of the following subsets of  $\mathbb{R}^2$  with the usual vector addition and scalar multiplication are subspaces?

- i.  $W_1$  is the set of all vectors of the form  $\begin{bmatrix} x \\ y \end{bmatrix}$ , where  $x \leq 0$ .
- ii.  $W_2$  is the set of all vectors of the form  $\begin{bmatrix} x \\ y \end{bmatrix}$ , where  $y \geq 0$
- iii.  $W_3$  is the set of all vectors of the form  $\begin{bmatrix} x \\ y \end{bmatrix}$ , where  $y = 0$

- A.  $W_1$  only.
- B.  $W_2$  only.
- C.  $W_3$  only.
- D.  $W_1$  and  $W_2$ .
- E. None of them.

Correct ans is C.