NAME:

Math 265

Midterm Exam 2 **Novemeber 5, 2013**

Instructions:

(1) No books, notes or calculators are allowed.
(2) If you need more room to write, write on the back of the page. DO NOT rip any pages apart from the test.
(3) Clearly indicate your answers for the multiple choice questions by writing in the correct answer in the space provided below the answer choices. If it is unclear what your answer choice is, I will mark it as incorrect.

Number	Points earned
#1	
#2	
#3	
#4	
#5	
#6	
#7	
#8	
#9	
#10	
#11	
#12	
Total	

1. If W is the subspace of $\mathbb{R}^5$ spanned by $\left\{ \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 0 \\ 2 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 3 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 3 \\ 5 \\ 9 \end{bmatrix} \right\}$, then $\dim W$ is

A. 0

B. 1

C. 2

D. 3

E. 4

Correct Answer is _____

2. Which of the following sets of vectors is linearly independent? *Hint: This question does not require extensive computations.*

A. $\begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$

B. $\begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$

C. $\begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}, \begin{bmatrix} 4 \\ 5 \\ 6 \end{bmatrix}, \begin{bmatrix} 7 \\ 8 \\ 10 \end{bmatrix}, \begin{bmatrix} 10 \\ 11 \\ 12 \end{bmatrix}$

D. $\begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}, \begin{bmatrix} 4 \\ 5 \\ 6 \end{bmatrix}, \begin{bmatrix} 7 \\ 8 \\ 10 \end{bmatrix}, \begin{bmatrix} 7 \\ 8 \\ 10 \end{bmatrix}$

E. $\begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 2 \\ 2 \\ 2 \end{bmatrix}, \begin{bmatrix} 3 \\ 3 \\ 3 \end{bmatrix}$

Correct Answer is _____

3. Let A be an m x n matrix. Which of the following is ALWAYS TRUE?

   i) If the rank of A is n, then the rank of A^T is m.

   ii) If $A\mathbf{x} = \mathbf{0}$ has a unique solution, then the nullity of A is 0.

   iii) If the rows of A are linearly independent, then the columns of A must be linearly independent.

   A. i) only

   B. i) and ii) only

   C. ii) only

   D. ii) and iii) only

   E. i), ii) and iii)

   Correct Answer is _____

4. The rank of the matrix $\begin{bmatrix} 1 & 1 & 1 & 1 \\ 2 & 2 & 2 & 2 \\ 3 & 3 & 3 & 3 \\ 4 & 4 & 4 & 4 \end{bmatrix}$ is

   A. 0

   B. 1

   C. 2

   D. 3

   E. 4

   Correct Answer is _____

5. Let A be a 4 x 7 matrix. If rank $A = 3$, then nullity A^T is

A. 4
B. 3
C. 1
D. 7
E. 0

Correct Answer is _____

6. Let $\mathbf{v}_1 = \begin{bmatrix} 2 \\ 2 \\ 2 \end{bmatrix}$, $\mathbf{v}_2 = \begin{bmatrix} 8 \\ 0 \\ 0 \end{bmatrix}$, $\mathbf{v}_3 = \begin{bmatrix} 10 \\ 2 \\ 2 \end{bmatrix}$, $\mathbf{v}_4 = \begin{bmatrix} 2 \\ 2 \\ 4 \end{bmatrix}$. Which of the following statements are true?

i) The set $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3, \mathbf{v}_4\}$ spans $\mathbb{R}^3$.
ii) The set $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3, \mathbf{v}_4\}$ is linearly dependent.
iii) The set $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3, \mathbf{v}_4\}$ is a basis for $\mathbb{R}^3$.

A. i) only
B. i) and ii) only
C. i) and iii) only
D. ii) only
E. i), ii) and iii)

Correct Answer is _____

7. Let $A = \begin{bmatrix} 2 & 0 & 4 & 6 & 1 \\ 1 & 1 & 4 & 2 & -1 \\ 3 & 1 & 8 & 8 & 0 \\ 6 & 6 & 24 & 12 & 10 \end{bmatrix}$. Its RREF is $\begin{bmatrix} 1 & 0 & 2 & 3 & 0 \\ 0 & 1 & 2 & -1 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$.

7a. Give a basis for Row A.

7b. Give a basis for Col A.

7c. Give a basis for Null A.

8. Let W be the subspace of $\mathbb{R}^4$ consisting of all vectors of the form $\begin{bmatrix} a \\ b+c \\ c \\ a \end{bmatrix}$. What is $\dim W^{\perp}$?

A. 4
B. 3
C. 2
D. 1
E. 0

Correct Answer is _____

9. Let $S = \begin{bmatrix} 1 \\ 0 \\ -2 \end{bmatrix}, \begin{bmatrix} 2 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 4 \\ -10 \\ 2 \end{bmatrix}$. Then S is

A. neither orthogonal, nor orthonormal.
B. orthogonal, but not orthonormal.
C. orthonormal.

Correct Answer is _____

10. Let W be the subspace of $\mathbb{R}^3$ with basis $\{\mathbf{u}_1, \mathbf{u}_2, \mathbf{u}_3\}$ where $\mathbf{u}_1 = \begin{bmatrix} 2 \\ 0 \\ 0 \end{bmatrix}$, $\mathbf{u}_2 = \begin{bmatrix} -2 \\ 1 \\ 0 \end{bmatrix}$ and $\mathbf{u}_3 = \begin{bmatrix} 0 \\ 2 \\ 2 \end{bmatrix}$. If we apply the Gram-Schmidt process to transform the vectors into an orthonormal basis $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$ for W, then $\mathbf{v}_3$ is

A. $\begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}$

B. $\begin{bmatrix} 0 \\ \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} \end{bmatrix}$

C. $\begin{bmatrix} 0 \\ 0 \\ 2 \end{bmatrix}$

D. $\begin{bmatrix} 0 \\ 0 \\ 3 \end{bmatrix}$

E. $\begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$

Correct Answer is _____

11. Let W be the subspace of $\mathbb{R}^3$ with basis $\left\{ \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} \right\}$. If $\mathbf{b} = \begin{bmatrix} 2 \\ -1 \\ 0 \end{bmatrix}$, then $\text{proj}_W \mathbf{b}$ is

A. $\begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix}$

B. $\begin{bmatrix} \frac{2}{\sqrt{2}} \\ -1 \\ \frac{2}{\sqrt{2}} \end{bmatrix}$

C. $\begin{bmatrix} 2 \\ -1 \\ 2 \end{bmatrix}$

D. $\begin{bmatrix} 2 \\ 1 \\ 2 \end{bmatrix}$

E. $\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$

Correct Answer is _____

12. Let A be an n x n *invertible* matrix. Which of the following is ALWAYS TRUE?

i) $A\mathbf{x} = \mathbf{b}$ has a unique solution for every $\mathbf{b} \in \mathbb{R}^n$.

ii) The rank of A is n.

iii) The rows of A form a basis for $\mathbb{R}_n$.

A. i) only

B. i) and ii) only

C. ii) only

D. ii) and iii) only

E. i), ii) and iii)

Correct Answer is _____