Math 265
Professor Priyam Patel
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Topics for Midterm 2

The following is a list of topics that will be covered on the second midterm. The list is not meant to be exhaustive, but should help you remember some of the topics we have covered since the last exam. You should certainly go through your homework problems, problems we did in class (especially from the handouts), the review problems I will post on the course pages etc.

- Understand vector spaces and subspaces. Though this was covered on the last exam, everything we have done since then has built on the concept of a vector subspace.
- Span: you should be able to define the span of a set of vectors, be able to tell when a set of vectors is a spanning set for a given subspace and be able to tell when a vector is the span of some given set of vectors.
- Linear independence and dependence: You should be able to define what it means for a set of vectors to be linearly independent or dependent. You should be able to use various methods to determine whether a set of vectors is linearly dependent or linearly independent. You should understand that if the zero vector is in a set of vectors, then that set is linearly dependent and why that is the case.
- Basis and Dimension: You should be able to define basis and dimension of a vector space. You should be able to produce a basis when you are given a spanning set for a vector space or subspace and use that basis to conclude what the dimension of the space is. You should be able to produce a basis for a subspace even when you are not given a spanning set to start out with.
- Col A, Row A and Null A along with rank and nullity: You should be able to define what the column space, row space and null space of a matrix are. You should be able to define the rank and nullity of a matrix and understand why rank $A+$ nullity $A = n$ (where A is an m x n matrix). You should understand the identities dealing with rank and nullity and the transpose of a matrix. You should be able to tell me the possible range of values for the rank of a matrix when I give you its size. In general, you should be able to say something about the RREF of the matrix when I give you the rank. You should be able to find a basis for Col A, Row A and Null A.
- You should understand the implications when rank $A =$ n, especially when A is an n x n matrix (A is then invertible, row equivalent to the identity, etc.). You should also understand the what this tells us about the column and row vectors of A (linearly independent).

- You should be able to compute expressions with dot products of vectors using the rules of dot products. You should be able to tell me when a given set of vectors is orthogonal or orthonormal using dot products and norms. You should also be able to define what is means for a set to be an orthogonal/orthonormal set or basis.
- You should be able to use the Gram-Schmidt process to produce an orthogonal basis for a subspace and then produce an orthonormal one by normalizing the vectors.
- You should be able to define the orthogonal complement of a set or subspace. You should be able to produce a basis for $W^{\perp}$ as well. You should be able to compute orthogonal projections of vectors onto subspaces and then use this to give the distance from the vector to the subspace.
- Given an m x n matrix A and a vector $\mathbf{b} \in \mathbb{R}^m$ you should be able to produce the least squares solution after you check that the rank of A is n.