Math 265
Professor Priyam Patel
11/4/13

Midterm 2 Practice Problems Answers

1. Yes. If you form the matrix $A = [\mathbf{u}_1\ \ \mathbf{u}_2\ \ \mathbf{u}_3]$ and row reduce you will find that rank $A = 3$. Therefore, the columns of A and thus the set $S = \{\mathbf{u}_1, \mathbf{u}_2, \mathbf{u}_3\}$ are linearly independent. Since S consists of 3 vectors and $\dim \mathbb{R}^3 = 3$ we can then conclude that S spans $\mathbb{R}^3$.

2. Yes, A is in the span of those matrices. Check this by transforming each matrix into a 4 x 1 vector and setting up an augmented matrix and row reducing. I computed the coefficients to be $\begin{bmatrix} 5 & 1 \\ -1 & 9 \end{bmatrix} = 2\begin{bmatrix} 1 & -1 \\ 0 & 3 \end{bmatrix} + 1\begin{bmatrix} 1 & 1 \\ 0 & 2 \end{bmatrix} + 1\begin{bmatrix} 2 & 2 \\ -1 & 1 \end{bmatrix}$. Since A can be written as a linear combination of the 3 matrices, the set of all four matrices cannot be linearly independent.

3. Linearly dependent.

4. All $c \neq 1$ will make the set of vectors linearly independent.

5. A basis is $\left\{ \begin{bmatrix} 1 \\ -2 \\ 0 \\ 2 \end{bmatrix}, \begin{bmatrix} 2 \\ -3 \\ 1 \\ -1 \end{bmatrix}, \begin{bmatrix} 2 \\ -2 \\ 3 \\ 0 \end{bmatrix} \right\}$.

6. A basis is $\left\{ \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix} \right\}$.

7. A basis for V is $\{t^3 + t^2 + 2t + 1, t^3 - 3t + 1, t^2 + t + 2, t + 1\}$.

8. A basis for W is $\left\{ \begin{bmatrix} -5 \\ 3 \\ 0 \end{bmatrix}, \begin{bmatrix} -7 \\ 0 \\ 3 \end{bmatrix} \right\}$ Note: your basis can be different than this! $\dim W = 2$. $\dim W^\perp = 1$. A basis for $W^\perp$ is $\begin{bmatrix} 3 \\ 5 \\ 7 \end{bmatrix}$.

9. $\dim W + \dim W^\perp = n$.

10. A basis is $\left\{ \begin{bmatrix} 1 \\ 2 \\ 2 \end{bmatrix}, \begin{bmatrix} 3 \\ 2 \\ 1 \end{bmatrix} \right\}$. A basis for $V^\perp$ is $\left\{ \begin{bmatrix} 2 \\ -5 \\ 4 \end{bmatrix} \right\}$.

11. For Null A $k = n$, for Row A $k = n$ (technically this is a subspace of $\mathbb{R}_n$) and for Col A $k = m$.

12. Rank is 3 and nullity is 1.

13. Is A is a 5 x 7 matrix, what are the possible values for rank A? rank A can take any values between 0 and 5.

If A is a 7 x 3 matrix, what are the possible values of rank A? rank A can take any values between 0 and 3.

If A is a 4 x 6 matrix, what are the possible values for nullity A? nullity A can take any values between 2 and 6.

14. A basis for Row A is $\{[1 \;\; -2 \;\; 0 \;\; 0], [0 \;\; 0 \;\; 1 \;\; 0], [0 \;\; 0 \;\; 0 \;\; 1]\}$. A basis for Null A is $\begin{bmatrix} 2 \\ 1 \\ 0 \\ 0 \end{bmatrix}$. A basis for Col A is $\left\{ \begin{bmatrix} 2 \\ 2 \\ 4 \\ 0 \end{bmatrix}, \begin{bmatrix} 2 \\ 3 \\ 3 \\ -1 \end{bmatrix}, \begin{bmatrix} -2 \\ -4 \\ -2 \\ 1 \end{bmatrix} \right\}$. rank $A = 3$, nullity $A = 1$, rank $A^T = 3$, nullity $A^T = 1$. Note: nullity A = nullity A^T is only because A is sqaure!

15. nullity $A^T = 1$

16. $\mathbf{u} \cdot \mathbf{v} = -5$ and $(\mathbf{u} + 2\mathbf{w}) \cdot (3\mathbf{v} - \mathbf{w}) = -105$.

17. $a = -2$ and $b = 2$. No $\{\mathbf{v}, \mathbf{u}, \mathbf{w}\}$ does not form an orthogonal set because $\mathbf{u}$ and $\mathbf{w}$ are not orthogonal.

18. $\frac{1}{\sqrt{6}} \begin{bmatrix} 2 \\ 1 \\ 1 \end{bmatrix}$ and $\frac{1}{\sqrt{2}} \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}$.

19. An orthogonal basis for the subspace is $\begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} -1 \\ 1 \\ 2 \end{bmatrix}$.

20. A basis for $V^\perp$ is $\begin{bmatrix} 2 \\ -2 \\ 1 \end{bmatrix}$. For $\mathbf{b} = \begin{bmatrix} 4 \\ 3 \\ 1 \end{bmatrix}$, $\text{proj}_V \mathbf{b} = \begin{bmatrix} 10/3 \\ 11/3 \\ 2/3 \end{bmatrix}$ and the distance from $\mathbf{b}$ to V is 1.

21. Orthogonal basis is $\left\{ \begin{bmatrix} 1 \\ -2 \\ 3 \end{bmatrix}, \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix}, \begin{bmatrix} -4 \\ 1 \\ 2 \end{bmatrix} \right\}$. Orthonormal basis is $\left\{ \frac{1}{\sqrt{14}} \begin{bmatrix} 1 \\ -2 \\ 3 \end{bmatrix}, \frac{1}{\sqrt{6}} \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix}, \frac{1}{\sqrt{21}} \begin{bmatrix} -4 \\ 1 \\ 2 \end{bmatrix} \right\}$
.

22. If A is an 4 x 4 matrix where rank $A = 4$, what can you say about det A? det $A \neq 0$.

    Do the columns of A form a basis for $\mathbb{R}^4$? Yes!

    How many non-zero rows are there in the RREF of A? There are 4.

    Does the system $A\mathbf{x} = \mathbf{b}$ have a solution for every $\mathbf{b} \in \mathbb{R}^4$? Yes.

    If so, is the solution unique? Yes

23. If A is a 6 x 6 matrix where rank $A = 4$, what can you say about det A? det $A = 0$.

    Do the rows of A form a basis for $\mathbb{R}_4$? This is a typo! The rows aren't even in $\mathbb{R}_4$, they are in $\mathbb{R}_6$. They cannot form a basis for $\mathbb{R}_6$.

    What is the dimension of Col A^T? 4

    How many vectors are there in any basis for Null A? 2

    How many solutions are there to the homogeneous system $A\mathbf{x} = \mathbf{0}$? infinitely many

24. If A is a 4 x 4 matrix and det $A = -3$ what is rank A? 4

25. If A is a 3 x 3 matrix and det $A = 0$, can you determine whether the column vectors of A form a basis for $\mathbb{R}^3$? No they cannot form a basis because rank A cannot be 3.

    What are the possible values for rank A? The possible values are 0, 1 and 2.