Math 265
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11/2/13

Midterm 2 Practice Problems

You should look over the Midterm 2 Topics Review, the multiple choice problems I listed on the website and also use these problems as practice. Homework problems and handout problems are also good practice problems. You should be prepared to answer both computational and conceptual problems (I have given a few examples of conceptual problems at the end of this document).

1. Suppose that $\mathbf{u}_1 = \begin{bmatrix} 1 \\ 3 \\ -2 \end{bmatrix}$, $\mathbf{u}_1 = \begin{bmatrix} 3 \\ 5 \\ -3 \end{bmatrix}$ and $\mathbf{u}_3 = \begin{bmatrix} 1 \\ -1 \\ 2 \end{bmatrix}$. Is the span of the set of vectors $S = \{\mathbf{u}_1, \mathbf{u}_2, \mathbf{u}_3\}$ all of $\mathbb{R}^3$? Justify your answer.

2. Is $A = \begin{bmatrix} 5 & 1 \\ -1 & 9 \end{bmatrix}$ in span $\left\{ \begin{bmatrix} 1 & -1 \\ 0 & 3 \end{bmatrix}, \begin{bmatrix} 1 & 1 \\ 0 & 2 \end{bmatrix}, \begin{bmatrix} 2 & 2 \\ -1 & 1 \end{bmatrix} \right\}$? Are these four matrices linearly dependent or linearly independent?

3. Are the vectors $\mathbf{u}_1 = \begin{bmatrix} 1 \\ 3 \\ -2 \end{bmatrix}$, $\mathbf{u}_1 = \begin{bmatrix} 3 \\ 5 \\ -3 \end{bmatrix}$ and $\mathbf{u}_3 = \begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix}$ linearly dependent or independent?

4. For what values of c are the vectors $\begin{bmatrix} -1 \\ 0 \\ -1 \end{bmatrix}$, $\begin{bmatrix} 2 \\ 1 \\ 2 \end{bmatrix}$ and $\begin{bmatrix} 1 \\ 1 \\ c \end{bmatrix}$ linearly independent?

5. Find a basis for span$\{\mathbf{w}_1, \mathbf{w}_2, \mathbf{w}_3, \mathbf{w}_4\}$ where

$$\mathbf{w}_1 = \begin{bmatrix} 1 \\ -2 \\ 0 \\ 2 \end{bmatrix}, \mathbf{w}_2 = \begin{bmatrix} 2 \\ -3 \\ 1 \\ -1 \end{bmatrix}, \mathbf{w}_3 = \begin{bmatrix} 0 \\ 1 \\ 1 \\ -5 \end{bmatrix}, \mathbf{w}_4 = \begin{bmatrix} 2 \\ -2 \\ 3 \\ 0 \end{bmatrix}.$$

6. Find a basis for all 2 x 2 symmetric matrices.

7. Find a basis for the subspace $W = \text{span}\{t^3+t^2+2t+1, t^3-3t+1, t^2+t+2, t+1, t^3+1\}$ of P_3.

8. Find a basis for the subspace $W = \left\{ \begin{bmatrix} x \\ y \\ z \end{bmatrix} : 3x + 5y + 7z = 0 \right\}$ of $\mathbb{R}^3$. What is dim W? What is dim $W^\perp$? Find a basis for $W^\perp$.

9. If W is a subspace of $\mathbb{R}^n$ then what is dim W + dim $W^\perp$ = ?

10. Find a basis for $V = \text{span} \left\{ \begin{bmatrix} 1 \\ 2 \\ 2 \end{bmatrix}, \begin{bmatrix} 3 \\ 2 \\ 1 \end{bmatrix}, \begin{bmatrix} 11 \\ 10 \\ 7 \end{bmatrix}, \begin{bmatrix} 7 \\ 6 \\ 4 \end{bmatrix} \right\}$. Find a basis for $V^\perp$.

Hint: The RREF of $\begin{bmatrix} 1 & 3 & 11 & 7 \\ 2 & 2 & 10 & 6 \\ 2 & 1 & 7 & 4 \end{bmatrix}$ is $\begin{bmatrix} 1 & 0 & 2 & 1 \\ 0 & 1 & 3 & 2 \\ 0 & 0 & 0 & 0 \end{bmatrix}$.

11. Suppose that A is an m x n matrix. For what value of k is Null A a subspace of $\mathbb{R}^k$? For what value of k is Row A a subspace of $\mathbb{R}^k$? For what value of k is Col A a subspace of $\mathbb{R}^k$?

12. Find the rank and nulity of the following matrix:

$$\begin{bmatrix} 1 & -1 & 2 & 3 \\ 2 & 6 & -8 & 1 \\ 2 & -1 & 0 & 1 \end{bmatrix}$$

13. Is A is a 5 x 7 matrix, what are the possible values for rank A? If A is a 7 x 3 matrix, what are the possible values of rank A? If A is a 4 x 6 matrix, what are the possible values for nullity A?

14. Given that A has RREF R, where

$$A = \begin{bmatrix} 2 & -4 & 2 & -2 \\ 2 & -4 & 3 & -4 \\ 4 & -8 & 3 & -2 \\ 0 & 0 & -1 & 1 \end{bmatrix} \quad \text{and} \quad R = \begin{bmatrix} 1 & -2 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix},$$

find bases for the row space of A, the column space of A and the null space of A. Give the rank and nullity of A and A^T.

15. If A is a 5 x 7 matrix and nulllity $A = 3$, what is nullity A^T?

16. Given that $\mathbf{u} = \begin{bmatrix} 1 \\ 3 \\ 2 \end{bmatrix}$, $\mathbf{v} = \begin{bmatrix} -2 \\ 1 \\ -3 \end{bmatrix}$ and $\mathbf{w}$ is a vector in $\mathbb{R}^3$ with $\|\mathbf{w}\| = 5$, $\mathbf{w} \cdot \mathbf{u} = 4$, and $\mathbf{w} \cdot \mathbf{v} = -6$, compute $\mathbf{u} \cdot \mathbf{v}$ and $(\mathbf{u} + 2\mathbf{w}) \cdot (3\mathbf{v} - \mathbf{w})$.

17. For what values of a and b is $\mathbf{v} = \begin{bmatrix} a \\ b \\ 2 \end{bmatrix}$ orthogonal to both $\mathbf{w} = \begin{bmatrix} 2 \\ 1 \\ 1 \end{bmatrix}$ and $\mathbf{u} = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}$. For those values of a and b does $\{\mathbf{v}, \mathbf{u}, \mathbf{w}\}$ form an orthogonal set?

18. Normalize the following vectors: $\begin{bmatrix} 2 \\ 1 \\ 1 \end{bmatrix}$ and $\begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}$.

19. Find an orthogonal basis for the subspace of $\mathbb{R}^3$ consisting of all vectors of the form $\begin{bmatrix} a \\ a+b \\ b \end{bmatrix}$.

20. Let V be the subspace of $\mathbb{R}^3$ spanned by $\left\{ \begin{bmatrix} 2 \\ 1 \\ -2 \end{bmatrix}, \begin{bmatrix} 1 \\ 2 \\ 2 \end{bmatrix} \right\}$. Find a basis for $V^\perp$. For $\mathbf{b} = \begin{bmatrix} 4 \\ 3 \\ 1 \end{bmatrix}$, find $\text{proj}_V \mathbf{b}$ and the distance from $\mathbf{b}$ to V.

21. Let $V = \text{span} \left\{ \begin{bmatrix} 1 \\ -2 \\ 3 \end{bmatrix}, \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix}, \begin{bmatrix} -3 \\ 3 \\ 3 \end{bmatrix} \right\}$. Using the Gram-Schmidt process, find an orthogonal basis for V and then transform that basis into an orthonormal one.

22. If A is an 4 x 4 matrix where rank $A = 4$, what can you say about det A? Do the columns of A form a basis for $\mathbb{R}^4$? How many non-zero rows are there in the RREF of A? Does the system $A\mathbf{x} = \mathbf{b}$ have a solution for every $\mathbf{b} \in \mathbb{R}^4$? If so, is the solution unique?

23. If A is a 6 x 6 matrix where rank $A = 4$, what can you say about det A? Do the rows of A form a basis for $\mathbb{R}_4$? What is the dimension of Col A^T? How many vectors are there in any basis for Null A? How many solutions are there to the homogeneous system $A\mathbf{x} = \mathbf{0}$?

24. If A is a 4 x 4 matrix and det $A = -3$ what is rank A?

25. If A is a 3 x 3 matrix and det $A = 0$, can you determine whether the column vectors of A form a basis for $\mathbb{R}^3$? What are the possible values for rank A?