**NAME:**

# Math 265

**Midterm Exam 1** **October 1, 2013**

**Instructions:**
(1) Show your work. No credit will be given for unsupported answers to problems requiring computation. You may receive credit for partially correct work even if your final answer is incorrect.
(2) You must use methods described thus far in the course when answering each question.
(3) No books, notes or calculators are allowed.
(4) If you need more room to write, write on the back of the page. DO NOT rip any pages apart from the test.

| Number | Points earned |
|---|---|
| #1 | |
| #2 | |
| #3 | |
| #4 | |
| #5 | |
| #6 | |
| #7 | |
| #8 | |
| Total | |

1. (a) Calculate the determinant of the following matrix.

$$A = \begin{bmatrix} 1 & -1 & 1 \\ 1 & -2 & 0 \\ 2 & -3 & 2 \end{bmatrix}$$

det $A =$

(b) Is $A$ invertible? Explain.

2. Consider the following system of linear equations:

$$\begin{aligned} x_1 + 0x_2 + x_3 &= 2 \\ 2x_1 + x_2 + 4x_3 &= 5 \\ x_1 + 0x_2 + 3x_3 &= 6 \end{aligned}$$

Use Gaussian elimination to find the the general solution to the system. Please indicate EVERY elementary row operation you use and write your solution in **vector form**.

3. Let $A$ and $B$ be $4 \times 3$ matrices. Suppose $B$ is obtained from $A$ using the following elementary row operations:

**1.** $R_1 \leftrightarrow R_2$

**2.** $2R_1 + R_2 \rightarrow R_2$

**3.** $4R_3 \rightarrow R_3$

Give the elementary matrix for each elementary row operation above:

**1.** $R_1 \leftrightarrow R_2$

**2.** $2R_1 + R_2 \rightarrow R_2$

**3.** $4R_3 \rightarrow R_3$

4. Determine whether or not the following subset of $\mathbb{R}^3$ is a **subspace** of $\mathbb{R}^3$. Show all of your work and explain your answer.

$$W = \left\{ \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} \in \mathbb{R}^3 : \; x_1 - 3x_2 + 5x_3 = 0 \right\}$$

5. Give an example of a matrix satisfying each of the following properties. If no such matrix exists please explain why.

   (a) A $4 \times 3$ matrix $R$ in reduced row echelon form such that $R\mathbf{x} = \mathbf{0}$ has exactly one solution.

   (b) A $3 \times 2$ matrix $A$ such that the subspace null $A = \{\mathbf{0}\}$, and for every $\mathbf{b} \in \mathbb{R}^3$ the equation $A\mathbf{x} = \mathbf{b}$ has at least one solution.

6. Let $A$ be an $n$ x $n$ nonsingular matrix. Which of the following statements must be TRUE?

   i) $A\mathbf{x} = \mathbf{0}$ has infinitely many solutions.
   ii) $A$ must be row equivalent to the identity matrix.
   iii) The system $A\mathbf{x} = \mathbf{b}$ always has a solution for all $\mathbf{b} \in \mathbb{R}^n$.
   iv) det $A \neq 0$.

   (a) All are true.
   (b) i), ii) and iii) only
   (c) iii) and iv) only
   (d) ii), iii) and iv) only
   (e) i), iii) and iv) only

7. If $A$ is a 3 x 3 matrix with det $A = 6$ and $B = 2A$, what is $\det(A^T B^{-1})$?

   (a) 1/2
   (b) 72
   (c) 48
   (d) 1/8
   (e) 1/72

8. Which of the following statements are always TRUE?

   i) If a linear system $A\mathbf{x} = \mathbf{b}$ has $m$ equations and $n$ variables and $m < n$, then the system has infinitely many solutions.
   ii) If $A$ and $B$ are $n$ x $n$ matrices and $AB$ is nonsingular (invertible) then both $A$ and $B$ must be nonsingular.
   iii) If $A$, $B$ and $C$ are $n$ x $n$ matrices such that $AB = AC$, then $B = C$.

   (a) i) only
   (b) ii) only
   (c) i) and ii) only
   (d) ii) and iii) only
   (e) i), ii) and iii)