Math 265
Professor Priyam Patel
1/14/16

Class Handout #1

From last class:

Legal moves in the method of elimination for solving linear systems:

- interchange equation i and equation j
- multiply an equation by a **nonzero** scalar
- replace the j^{th} equation with c times equation i plus equation j

A system of linear equations can have:

- no solution
- a unique solution
- infinitely many solutions

Section 1.2: Matrices

Definition: An $m \times n$ matrix is a rectangular array of $m \cdot n$ real (or complex) numbers arranged in m horizontal rows and n vertical columns:

The i^{th} row of A is $\text{row}_i(A) = \begin{bmatrix} a_{i1} & a_{i2} & \cdots & a_{in} \end{bmatrix}$.

The j^{th} column of A is $\text{col}_j(A) = \begin{bmatrix} a_{1j} \\ a_{2j} \\ \vdots \\ a_{mj} \end{bmatrix}$

When $m = n$ we say that A is **square** and that the numbers $a_{11}, a_{22}, \ldots, a_{nn}$ form the **main diagonal** of A.

The entry a_{ij} is called the i, j^{th} element of A or the (i, j)–entry. (i^{th} row and j^{th} column).

Sometimes we use the shorthand $A = [a_{ij}]$.

Exercise 1: Let

$$A = \begin{bmatrix} 1 & 2 & 3 \\ -1 & 0 & 1 \end{bmatrix}, \; B = \begin{bmatrix} 1+i & 4i \\ 2-3i & -3 \end{bmatrix}, \; C = \begin{bmatrix} 3 \end{bmatrix}, \; D = \begin{bmatrix} -1 & 0 & 2 \end{bmatrix}, \; E = \begin{bmatrix} 1 \\ 1 \\ 2 \end{bmatrix}.$$

The size of A is , and $a_{12} =$.

The size of B is , and $b_{22} =$.

The size of C is .

The size of D is , and $d_{12} =$.

The size of E is , and $e_{31} =$.

Definition: An $n \times 1$ matrix is also called an n–vector and denoted by $\mathbf{u} = \begin{bmatrix} u_1 \\ u_2 \\ \vdots \\ u_n \end{bmatrix}$.

Definition: The set of all n–vectors with real entries is denoted R^n. The set of all n–vectors with complex entries is denoted C^n.

Definition: Two $m \times n$ matrices A and B are **equal** if they are equal entry-wise, that is, $a_{ij} = b_{ij}$ for all i and j.

<u>Matrix Addition:</u>

Definition: If $A = [a_{ij}]$ and $B = [b_{ij}]$ are two $m \times n$ matrices, then $A + B$ is an $m \times n$ matrix $C = [c_{ij}]$ such that $c_{ij} = a_{ij} + b_{ij}$ for all i and j. (add matrices entry-wise)

So if $A = \begin{bmatrix} 1 & -2 & 3 \\ 2 & -1 & 4 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 0 & 4 \\ 3 & 2 & 0 \end{bmatrix}$, $A + B = \begin{bmatrix} & & & & \\ & & & & \end{bmatrix}$

**Note: The sum $A + B$ is only defined when A and B are of the same size.

Note: If A is an $m \times n$ matrix and O is the $m \times n$ **zero matrix, then $A + O = A$. In particular, if $\mathbf{x}$ is an n–vector, then $\mathbf{x} + \mathbf{0} = \mathbf{x}$, where $\mathbf{0}$ is the n–vector all of whose entries are zero (the **zero vector**).

Scalar Multiplication:

Definition: If $A = [a_{ij}]$ is an $m \times n$ matrix and r is a real scalar, then the **scalar multiple** of A by r, denoted by rA, is the $m \times n$ matrix whose (i, j)–entry is $r \cdot a_{ij}$. (scale every entry)

Definition: If $A_1, A_2, \ldots, A_k$ are $m \times n$ matrices and $c_1, c_2, \ldots, c_k$ are real scalars, then an expression of the form

$$c_1A_1 + c_2A_2 + \cdots + c_kA_k$$

is called a **linear combination** of $A_1, \ldots, A_k$. The $c_1, \ldots, c_k$ are the **coefficients**.

Can also be written:

$$\sum_{i=1}^{k} c_iA_i = c_1A_1 + c_2A_2 + \cdots + c_kA_k$$

Definition: If $A = [a_{ij}]$ is an $m \times n$ matrix, then the **transpose** of A, denoted by $A^T = [a^T_{ij}]$, is the $\underline{n \times m}$ matrix defined by $a^T_{ij} = a_{ji}$. (The (i, j)–entry of A^T is equal to the (j, i)–entry of A.)