MATH 351 FINAL REVIEW – CONCEPTS

1. Rough outline of what we have covered

- Matrices: size of matrix, adding matrices (when this is defined), scalar multiple of matrix, transpose of a matrix
- $\mathbb{R}^n$ as the set of all n-vectors
- Linear combinations, linear dependence and independence
- Span and geometric concept of what the span of one or two vectors in $\mathbb{R}^n$ looks like
- Vector Space Properties and the vector spaces $\mathbb{R}^n$, $M(m, n)$, and P_n
- Systems of linear equations: equivalent systems, quickly determining whether a proposed solution to a linear system is in fact a solution, types of solutions (one, none, inf many), parametric form of the solution set and the geometric interpretation of the solution set using parametric form, consistent and inconsistent systems, homogeneous systems, rank of a system, augmented and coefficient matrix for a system
- Gaussian Elimination: elementary row operations, row equivalent matrices, EF, RREF, pivot variables, free variables, the "More Unknowns Theorem"
- Checking whether vector is in span of some others when working in the vector spaces $\mathbb{R}^n$, P_n, $M(m, n)$
- Checking whether a set of vectors spans the whole vector space $\mathbb{R}^n$, P_n or $M(m, n)$
- Subspaces
- Column space
- Null space
- Translation theorem
- Testing for linear dependence and independence
- Spans of linearly dependent sets and throwing out vectors–Proposition 1 Page 102
- Bases
- Dimension
- Dimension and basis theorem – Theorem 2 Page 116, Theorem 3 Page 118
- Row space
- Rank
- Nullity
- Rank-nullity theorem
- Rank and nullity in relation to transposes of matrices

- Rank of A and the solution set to the system Ax=b – Theorem 5, Theorem 7, Definition 4, Theorem 8 and Theorem 9 Pages 144-146
- Nonsingular matrix
- Nonsingular matrix theorem – Theorem 9 and Proposition 2 Pages 146-147
- Linear transformations and linearity properties
- Matrix transformations
- Obtaining the matrix A for a linear transformation $T : \mathbb{R}^n \longrightarrow \mathbb{R}^m$
- Identity transformation and identity matrix
- Matrix multiplication and composition of transformations
- Inverses of matrices and inverse transformations
- Invertibility and nonsingularity– Theorem 2 Page 195
- Coordinate vectors and the $\mathcal{P}_B$ and $\mathcal{C}_B$ matrices
- Determinants – Compute the determinant of a matrix via cofactor expansion (in small cases and when there is an obvious row to expand along). Compute the determinant of an upper triangular or lower triangular matrix. Compute the determinant of a matrix by using row operations to put it in upper triangular form. Understand the effect that row operations have on the determinant of a matrix. Quickly determine whether a matrix is invertible using determinant.
- Eigenvalues and eigenvectors, characteristic polynomials, multiplicity of eigenvalues, eigenspaces and finding their bases, diagonalizability of a matrix of $\mathbb{R}$ (see the outline below for the procedure for diagonalization).
- Norm/lengths of vectors in $\mathbb{R}^n$
- Distance and angle between two vectors in $\mathbb{R}^n$
- Checking whether vectors are orthogonal via their dot products, dot product calculations and properties in general
- Checking if a set of vectors is orthogonal (you should remember that an orthogonal set of vectors cannot have $\mathbf{0}$ in it)
- Simplified formula for coordinate vectors when given an orthogonal basis for $\mathbb{R}^n$ – Theorem 4 Page 318
- An orthogonal set of vectors in $\mathbb{R}^n$ is linearly independent – Theorem 5 page 319
- $S^{\perp}$ and orthogonal complements of subspaces of $\mathbb{R}^n$
- The projection $\text{Proj}_{\mathcal{W}}(Y)$ of a vector $Y \in \mathbb{R}^n$ onto a subspace $\mathcal{W}$ of $\mathbb{R}^n$ given an orthogonal basis for $\mathcal{W}$, and the calculation of $\text{Orth}_{\mathcal{W}}(Y) \in \mathcal{W}^{\perp}$ – Theorem 1 Page 329
- The Gram-Schmidt process to produce an orthogonal basis when you are given a spanning set of basis for a subspace of $\mathbb{R}^n$

Procedure for diagonalizing matrices:

(1) Find the eigenvalues of A, that is find the roots of the characteristic polynomial of A. If there are no complex eigenvalues then proceed to step 2. If there are any complex eigenvalues then A is not diagonalizable.
(2) Find the dimensions of the corresponding eigenspaces by finding dim $\text{Null}(A - \lambda_0 I_n)$ = nullity $(A - \lambda_0 I_n)$. If nullity $(A - \lambda_0 I_n)$ is equal to the multiplicity of λ_0 for each eigenvalue λ_0 then A is diagonalizable.
(3) To diagonalize A, start by finding a basis of eigenvectors for the eigenspace corresponding to each eigenvalue. That is, find a basis for $\text{Null}(A - \lambda_0 I_n)$.
(4) Form the invertible matrix Q whose column vectors are the eigenvectors you found in step 3. Form the diagonal matrix D whose entries along the main diagonal are the corresponding eigenvalues (the order of the values is determined by how you put down the eigenvectors as the columns of Q).

This list is not meant to be exhaustive. I would suggest reading through all of the sections of Penney we have covered, reviewing all definitions and making sure you understand why the Propositions and Theorems are true.