Chapter 12 + extra

Suppose we have proven that M has $\chi^{orb} > 0$ and $e \neq 0$ iff its a spherical mfd.

Let's try to understand these mfds and their SFS better.

$\chi(S^2) - \sum (1 - \frac{1}{p_i})$

9.2 Spherical manifolds according to their base orbifolds

We now further describe the structure of spherical 3-manifolds according on their base orbifolds. Since $\chi^{orb}B > 0$ there are three infinite families for B given by $S^2(n,n), S^2(2,2,n)$ and $\mathbb{R}P^2(n)$ with $n > 0$. The remaining possibilities for B are $S^2(2,3,n)$ with $n = 3,4$ or 5.

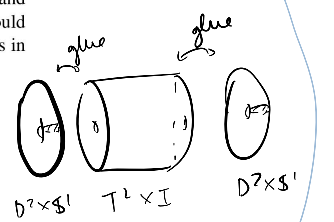
obtained from S^2 via quotient by a finite group of Isometries (finite subgroup of $O(3)$ – rotations about origin)

The two families with base orbifold $S^2(2,2,n)$ and $\mathbb{R}P^2(n)$ all have a Siefert fibering with base $S^2(2,2,n)$, see Hatcher (2.3)(d). Additionally, $S^2(n,n)$ is an orbifold cover of $S^2(2,2,n)$ of degree 2. If $B = S^2(n,n)$ then M is a *lens space* (defined below) add a reference here. Therefore, all manifolds M' with base $S^2(2,2,n)$ are *prism manifolds*, i.e. they have a two-fold covering that is a lens space M, induced by the orbifold cover. In particular, M' is obtained from M via a quotient by a free involution. We describe lens spaces first and then prism manifolds. Suppose $B = S^2(n,n)$ so that M is a lens space. Let A and C each be a solid torus, with meridians α and μ, respectively. The set of lens spaces is obtained by identifying ∂A to ∂C where the curves marked α in ∂A and μ in ∂C have the property that $\alpha \neq \mu$ def 10.1.4 and section 10.1.3 of Martelli. In the lens space, we will refer to $\partial A = \partial C$ as the central torus. We emphasize that the meridian of C is arbitrary subject to $\mu \neq \alpha$.

Now consider the case where M' is a prism manifold, corresponding to a quotient of a lens space M by a free involution τ. By taking a neighborhood of the central torus in the lens space M, we decompose M as

$$(D^2 \times S^1) \bigcup (T^2 \times [-1,1]) \bigcup (D^2 \times S^1),$$

where $T^2 \times \{0\}$ is the central torus in the lens space. Since τ is a free involution, it must swap the two solid tori in this decomposition. We let C denote the corresponding solid torus in M'. Letting $A = T^2 \times [-1,1]/\tau|_{T^2 \times [-1,1]}$, we have that A is the orientable twisted I-bundle over a Klein bottle so that ∂A is a torus, and the prism manifold M' is obtained by gluing A to the solid torus C along their boundary tori, with the following condition. Let α and β be the curves in ∂A representing the basis $\binom{1}{0}, \binom{0}{1}$ of the first homology of ∂A. The meridian μ in C should not be identified with α or β in ∂A. This condition is necessary to ensure that the two-fold cover M of M' is in fact a lens space and not $S^2 \times S^1$. we might want to add a reference here too if we can find one



= Lens space / τ

= $D^2 \times S^1 \sqcup K \tilde{\times} I$ / Dehn Fill

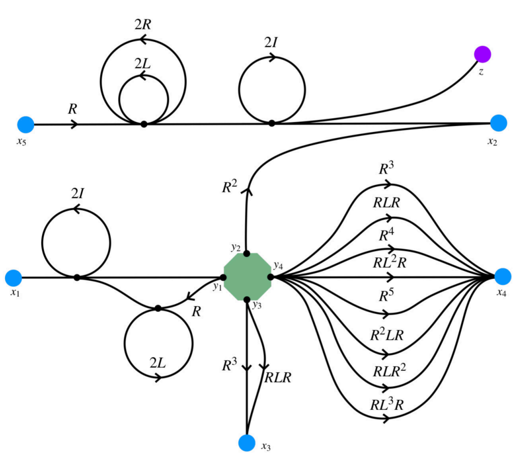
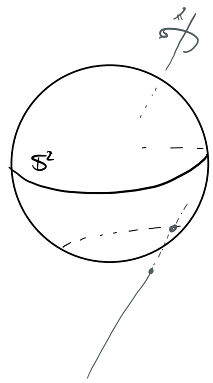
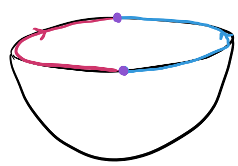
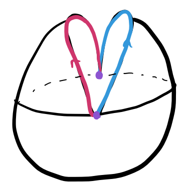


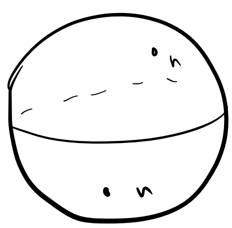
Figure 7: Branched manifold for Spherical Geometry



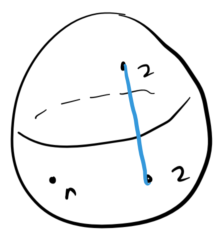
rotate by π

$\cong$  $=$  $= \mathbb{S}^2(2,2)$

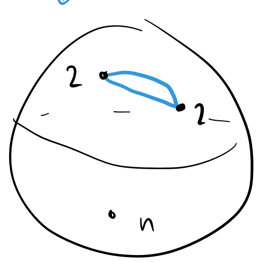
Same but rotation by $\frac{2\pi}{3}$ gives $\mathbb{S}^2(3,3)$



$\longrightarrow$

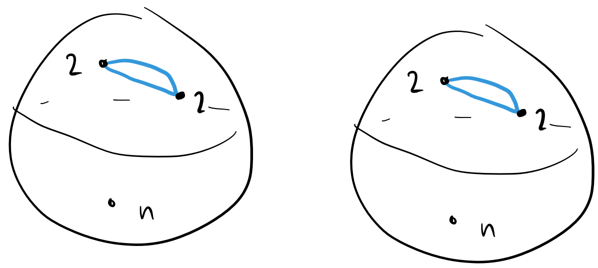


slice open



copy

paste

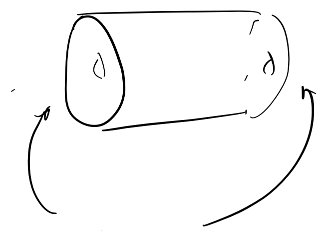


$$\begin{tikzcd} M \arrow[r] \arrow[d] & S^2(n,n) \arrow[d] \\ M' \arrow[r] & S^2(2,2,n) \end{tikzcd}$$

S^2

$\times S^1$

Cut out two solid tori



Dehn Fill