# Math 265 Midterm 1 Review

February 7, 2016

## 1 Important Concepts

The following is a list of some of the most important concepts we have covered so far. You should be able to give a complete, correct definition for each term. Even if you feel like you kinda, sorta, basically know the definition, challenge yourself to produce a precise statement of the definition.

- Elementary row operation
- Row echelon form
- Reduced row echelon form
- The cofactor of $a_{ij}$
- Inverse of a matrix
- The minor of $a_{ij}$
- Adjoint of a matrix
- Elementary matrix
- Upper triangular, lower triangular, and diagonal matrix
- Linear combination

## 2 Things you should be able to do

This list is not meant to be exhaustive, but to remind you of things I may ask you to do on the exam. These are roughly in the order they appear in the book.

- Add, scalar multiply, and transpose matrices, and evaluate expressions involving all of these elements. Recognize when such expressions are not defined.

- Multiply two matrices. Evaluate expressions involving matrix multiplication, as well as addition, scalar multiplication and transposes. Recognize when such products are undefined.
- Recognize a matrix in reduced row echelon form. Identify the pivot positions and pivot columns of a matrix.
- Use Gaussian Elimination to put a matrix into reduced row echelon form.
- Produce the general solution to a system of linear equations, if consistent, using an augmented matrix and Gaussian Elimination. Recognize if a linear system is inconsistent.
- Quickly determine whether a proposed solution to a linear system is in fact a solution.
- Write down the elementary matrix corresponding to a given elementary row operation.
- Compute the determinant of a matrix via cofactor expansion (in small cases and when there is an obvious row to expand along).
- Compute the determinant of an upper triangular or lower triangular matrix.
- Compute the determinant of a matrix by using row operations to put it in upper triangular form. Understand the effect that row operations have on the determinant of a matrix.
- Quickly determine whether a matrix is invertible.
- Quickly determine whether two matrices are inverses of each other.
- Evaluate expressions involving inverses along with matrix multiplication, addition, scalar multiplication, and transposes.
- Find the inverse of a matrix using the augmented matrix $[A\ I_n]$. Find the inverse of a matrix using the adjoint of the matrix. Find a specific entry in the inverse of a matrix using cofactors.
- Solve a system of linear equations using Cramer's rule.
- Quickly find the inverse of an elementary matrix. Recognize when a matrix is not invertible.

## 3 Questions to test your conceptual understanding

Please do not be scared off by this section of the review. These questions are meant to test whether or not you have really grasped the concepts we have covered. Do not worry, your exam will be a mix of conceptual and practical questions.

- For any linear system of equations, what are the possible number of solutions?
- How many types of elementary row operations are there and what are they?
- For matrices $A$, $B$, if $AB$ is defined must $BA$ also be defined?
- Is the matrix product $AA^T$ always defined?
- What effect do the different elementary row operations have on the determinant?
- What is the relationship between the determinant of a matrix and the determinant of its inverse?
- What is the relationship between the determinant of a matrix and the determinant of its transpose?
- Can a matrix have two different reduced row echelon forms?
- Is $\det(AB) = (\det A)(\det B)$? Is $\det(A+B) = (\det A) + (\det B)$?
- Which of the following exist for every matrix: a transpose, a reduced row echelon form, an inverse, a determinant? If one doesn't exist, is there anything we can say about when it does exist?
- Is every invertible matrix square? Is every square matrix invertible?
- Is the product of invertible matrices always invertible? Is the sum of invertible matrices always invertible?
- What can you say about the reduced row echelon form of an invertible matrix?
- What can you say about the reduced row echelon form of a non-invertible matrix?
- Is there a $2 \times 2$ matrix $A$ such that $A\mathbf{x} = \mathbf{0}$ has only $\mathbf{0}$ as a solution?
- Is there a $2 \times 2$ matrix $A$ such that $A\mathbf{x} = \mathbf{0}$ has infinitely many solutions?

- Is there a $2 \times 2$ matrix $A$ such that $A\mathbf{x} = \mathbf{0}$ has no solutions?
- If the product $AB$ is a zero matrix, must either $A$ or $B$ be a zero matrix?
- If $A$ is a $2 \times 2$ matrix, what is $\det(-A)$? What if $A$ is $3 \times 3$? If $A$ is a $2 \times 2$ matrix, what is $\det 2A$? What if $A$ is $3 \times 3$?
- For each of the following, give an example of a matrix satisfying the criteria, or explain why no such matrix exists:
  - a 3 x 4 matrix $R$ in RREF such that for *every* $\mathbf{c} \in \mathbb{R}^3$ the equation $R\mathbf{x} = \mathbf{c}$ has a solution $\mathbf{x} \in \mathbb{R}^4$
  - a 3 x 4 matrix $R$ in RREF such that the equation $R\mathbf{x} = \mathbf{0}$ has a *unique* solution $\mathbf{x} \in \mathbb{R}^4$
  - a 4 x 3 matrix $R$ in RREF such that for *every* $\mathbf{c} \in \mathbb{R}^4$ the equation $R\mathbf{x} = \mathbf{c}$ has a solution $\mathbf{x} \in \mathbb{R}^3$
  - a 4 x 3 matrix $R$ in RREF such that the equation $R\mathbf{x} = \mathbf{0}$ has a *unique* solution $\mathbf{x} \in \mathbb{R}^3$
  - a 4 x 4 matrix $R$ in RREF such that the equation $R\mathbf{x} = \mathbf{0}$ has a *nonzero* solution $\mathbf{x} \in \mathbb{R}^4$