Midterm Exam 2, Thus, Nov 5, 2015. Time: 60 min

Instructions: Write your name in the top left corner. There are ten multiple choice questions. Clearly **circle exactly one** *of the letters labelling the five alternatives. No credit will be given if there is* **any** *confusion about which answer you are choosing. So think carefully before circling. There is no negative marking – so attempt all questions.*

---

1. Let $A = \begin{pmatrix} 1 & 0 & 1 & 2 & 1 \\ 1 & 0 & 1 & 2 & 3 \\ 2 & 1 & 0 & 1 & 2 \\ -1 & -1 & 1 & 1 & 0 \end{pmatrix}$ Then the nullity of A is

   A. 0

   B. 1

   C. 2

   D. 3

   E. 4

2. Let

$$\mathbf{v}_1 = \begin{bmatrix} a^2 \\ 0 \\ 1 \end{bmatrix}, \mathbf{v}_2 = \begin{bmatrix} 0 \\ a \\ 2 \end{bmatrix}, \mathbf{v}_3 = \begin{bmatrix} 4 \\ 0 \\ 1 \end{bmatrix}.$$

   Find all values of a which $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$ is a linearly independent set?

   A. $a = \pm 2$ only

   B. $a = 0, \pm 1$ only

   C. any real number except $a = 0, \pm 2$

   D. any real number except $a = \pm 2$

   E. any real number except $a = 0$

3. Let

$$\mathbf{v}_1 = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \mathbf{v}_2 = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}, \mathbf{v}_3 = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}, \mathbf{v}_4 = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}, \mathbf{v}_5 = \begin{bmatrix} 3 \\ 2 \\ 1 \end{bmatrix}.$$

Which of the following is NOT true?

A. $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$ is a basis for $\mathbf{R}^3$

B. $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3, \mathbf{v}_4\}$ spans $\mathbf{R}^3$

C. $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3, \mathbf{v}_5\}$ is a linearly dependent set

D. $\{\mathbf{v}_3, \mathbf{v}_4, \mathbf{v}_5\}$ is a linearly independent set

E. $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3, \mathbf{v}_4, \mathbf{v}_5\}$ spans $\mathbf{R}^3$

4. Let W be a subspace of $\mathbf{R}^5$ spanned by

$$\begin{bmatrix} 1 \\ 5 \\ 9 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 2 \\ 6 \\ 10 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 3 \\ 7 \\ 11 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 4 \\ 8 \\ 12 \\ 0 \\ 1 \end{bmatrix}.$$

Which of the following is the dimension of W?

A. 1

B. 2

C. 3

D. 4

E. 5

5. Let A be a 4×5 matrix such that $\operatorname{rank} A = 4$. How many solutions does a linear system $A\mathbf{x} = \mathbf{b}$ have?

A. 0

B. 1

C. 2

D. infinitely many

E. Cannot be determined from the given information

6. How many different values are possible for the rank of a 6×4 matrix?

A. 4

B. 5

C. 6

D. 7

E. infinitely many

7. Define an inner product on P_2 by

$$(p(t), q(t)) = \int_0^1 p(t)q(t)\,dt.$$

Choose the constant a so that $a+x$ is orthogonal to x^2 with respect to this inner product.

A. $\frac{3}{4}$

B. $\frac{4}{3}$

C. $-\frac{3}{4}$

D. $-\frac{4}{3}$

E. None of the above

8. Let V be an inner product space and $\{\mathbf{u}, \mathbf{v}, \mathbf{w}\}$ be an orthonormal set of vectors. Then $||\mathbf{u} - 2\mathbf{v} - 3\mathbf{w}||^2$ equals

A. 0

B. 6

C. 14

D. 16

E. Cannot be determined by the given information.

9. Let V be an inner product space and $\mathbf{u}, \mathbf{v} \in V$. Which of the following is always true ?

A. $||\mathbf{u}+\mathbf{v}||^2 + ||\mathbf{u}-\mathbf{v}||^2 = 2||\mathbf{u}||^2 + 2||\mathbf{v}||^2$

B. $||\mathbf{u}|| + ||\mathbf{v}|| \leq ||\mathbf{u}+\mathbf{v}||$

C. $(\mathbf{u}, \mathbf{v}) = -(\mathbf{v}, \mathbf{u})$

D. $||\mathbf{u}|| ||\mathbf{v}|| \leq |(\mathbf{u}, \mathbf{v})|$

E. $||\mathbf{u}||^2 + ||\mathbf{v}|^2 = ||\mathbf{u}+\mathbf{v}||^2$

10. Let $V = \mathbb{R}^4$ with the dot product as inner product, and W the subspace defined by $X_1 - 2X_2 + 3X_3 - 4X_4 = 0$. Let $\mathbf{v} = [1, -2, 3, -4]^T$, and let $\mathbf{v} = \mathbf{w} + \mathbf{w}^\perp$ where $\mathbf{w} \in W$ and $\mathbf{w}^\perp \in W^\perp$. Which of the following is true ?

A. $\mathbf{w} = [3, 1, 1, 1]^T$.

B. $\mathbf{w}^\perp = [3, 1, 1, 1]^T$.

C. $\mathbf{w} = \mathbf{w}^\perp = [1/2, -1, 3/2, -2]^T$.

D. $\mathbf{w} = \mathbf{0}$.

E. $\mathbf{w}^\perp = \mathbf{0}$.