Math 265
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2/16/16

Class Handout # 8

Theorem 4.1 (Properties of n-vectors):

If $\mathbf{u}$, $\mathbf{v}$ and $\mathbf{w}$ are vectors in $\mathbb{R}^n$ and c and d are real scalars, then the following properties hold:

1. $\mathbf{u} + \mathbf{v} = \mathbf{v} + \mathbf{u}$
2. $(\mathbf{u} + \mathbf{v}) + \mathbf{w} = \mathbf{u} + (\mathbf{v} + \mathbf{w})$
3. There exists and element $\mathbf{0}$, the zero vector, such that $\mathbf{u} + \mathbf{0} = \mathbf{0} + \mathbf{u} = \mathbf{u}$
4. For every vector $\mathbf{u}$, there exists and element **-u** such that $\mathbf{u} + (-\mathbf{u}) = \mathbf{0}$
5. $c(\mathbf{u} + \mathbf{v}) = c\mathbf{u} + c\mathbf{v}$
6. $(c + d)\mathbf{u} = c\mathbf{u} + d\mathbf{u}$
7. $c(d\mathbf{u}) = (cd)\mathbf{u}$
8. $1\mathbf{u} = \mathbf{u}$

Recall that we also said the following properties hold for $\mathbb{R}^n$ (viewed as the set of all n-vectors):

(a) If $\mathbf{u}$ and $\mathbf{v}$ are n-vectors, then $\mathbf{u} + \mathbf{v}$ is an n-vector.

(b) If $\mathbf{u}$ is an n-vector and c is any real scalar, then $c\mathbf{u}$ is an n-vector.

Exercise 2: Let M_{mn} be the **set** of all m x n matrices with real entries. Do the properties (a) and (b) above hold for M_{mn}?

Hint: (a) should be restated in the context of M_{mn} as: If A and B are in M_{mn} (they are two m x n matrices), is $A + B$ in M_{mn}? Property (b) should be similarly restated.

Do properties 1 through 8 of Theorem 4.1 above hold for M_{mn}? What plays the role of the zero vector in property 3?

Exercise 3: Let P_n be the set of all polynomials of degree $\leq n$ together with the zero polynomial $0(x)$.

Do properties (a) and (b) above hold for P_n?

Do properties 1 through 8 of Theorem 4.1 above hold for P_n? What plays the role of the zero vector in property 3?

Exercise 4: Let P be the set of all polynomials of any degree together with the zero polynomial $0(x)$.

Do properties (a) and (b) above hold for P?

Do properties 1 through 8 of Theorem 4.1 above hold for P? What plays the role of the zero vector in property 3?

Exercise 5: Let $C(-\infty, \infty)$ be the set of all real valued continuous functions on $\mathbb{R}$.

Do properties (a) and (b) above hold for $C(-\infty, \infty)$?

Do properties 1 through 8 of Theorem 4.1 above hold for $C(-\infty, \infty)$? What plays the role of the zero vector in property 3?