Math 265
Professor Priyam Patel
2/4/16

Class Handout #7

Exercise 1:

$$A(\text{adj } A) = \begin{bmatrix} 3 & -2 & 1 \\ 5 & 6 & 2 \\ 1 & 0 & -3 \end{bmatrix} \begin{bmatrix} -18 & -6 & -10 \\ 17 & -10 & -1 \\ -6 & -2 & 28 \end{bmatrix} =$$

$$(\text{adj } A)A = \begin{bmatrix} -18 & -6 & -10 \\ 17 & -10 & -1 \\ -6 & -2 & 28 \end{bmatrix} \begin{bmatrix} 3 & -2 & 1 \\ 5 & 6 & 2 \\ 1 & 0 & -3 \end{bmatrix} =$$

Exercise 2:

Consider the linear system $A\mathbf{x} = \mathbf{b}$ where $A = \begin{bmatrix} 1 & 2 & 0 \\ 0 & 2 & -3 \\ 0 & -2 & 0 \end{bmatrix}$ and $\mathbf{b} = \begin{bmatrix} 1 \\ 4 \\ -3 \end{bmatrix}$.

Find the unique solution $\mathbf{x}$ using Cramer's Rule.

Theorem 4.1 (Properties of n-vectors):

If $\mathbf{u}$, $\mathbf{v}$ and $\mathbf{w}$ are vectors in $\mathbb{R}^n$ and c and d are real scalars, then the following properties hold:

1. $\mathbf{u} + \mathbf{v} = \mathbf{v} + \mathbf{u}$
2. $(\mathbf{u} + \mathbf{v}) + \mathbf{w} = \mathbf{u} + (\mathbf{v} + \mathbf{w})$
3. There exists and element $\mathbf{0}$, the zero vector, such that $\mathbf{u} + \mathbf{0} = \mathbf{0} + \mathbf{u} = \mathbf{u}$
4. For every vector $\mathbf{u}$, there exists and element **-u** such that $\mathbf{u} + (-\mathbf{u}) = \mathbf{0}$
5. $c(\mathbf{u} + \mathbf{v}) = c\mathbf{u} + c\mathbf{v}$
6. $(c + d)\mathbf{u} = c\mathbf{u} + d\mathbf{u}$
7. $c(d\mathbf{u}) = (cd)\mathbf{u}$
8. $1\mathbf{u} = \mathbf{u}$

Exercise 3: Do the following properties hold for $\mathbb{R}^n$ (viewed as the set of all n-vectors)?

(a) If $\mathbf{u}$ and $\mathbf{v}$ are n-vectors, then $\mathbf{u} + \mathbf{v}$ is an n-vector.

(b) If $\mathbf{u}$ is an n-vector and c is any real scalar, then $c\mathbf{u}$ is an n-vector.

Definition: A *real vector space* is a set of elements V on which there are two operations (addition and scalar multiplication) which obey properties 1-8 and (a) and (b) above.

Note: Property (a) is called **closed under addition** and property (b) is called **closed under scalar multiplication**.