Math 265
Professor Priyam Patel
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Class Handout #6

Exercise 1: What is $\det \begin{bmatrix} 2 & -4 & 13 \\ 0 & 3 & 2 \\ 0 & 0 & -1 \end{bmatrix}$?

Exercise 2:

Let A be a 4 x 4 matrix with $\det(A) = -2$.

- Describe the set of all solutions to the homogeneous system $A\mathbf{x} = \mathbf{0}$.
- What is the RREF of A?
- Can the linear system $A\mathbf{x} = \mathbf{b}$ have more than one solution? Explain.
- Does A^{-1} exist?
- Give an expression for a solution to the linear system $A\mathbf{x} = \mathbf{b}$.

Exercise 3: Let A be a square matrix with $\det(A) = 0$.

- What can you say about the RREF of A?
- Can the system $A\mathbf{x} = \mathbf{b}$ have one solution? Can it have infinitely many solutions? Can it have no solution? Explain.
- How many solutions does the system $A\mathbf{x} = \mathbf{0}$ have?
- Does A^{-1} exist?

What we have shown is that the following statements are *equivalent* for an nxn matrix A:

1. A is invertible (nonsingular).
2. $A\mathbf{x} = \mathbf{0}$ has only the trivial solution.
3. A is row equivalent to I_n. (The RREF of A is I_n.)
4. The linear system $A\mathbf{x} = \mathbf{b}$ has a unique solution for every n-vector $\mathbf{b}$.
5. A is a product of elementary matrices.
6. $\det(A) \neq 0$.

Example 1: Let $A = \begin{bmatrix} 1 & 2 & 3 \\ -2 & 3 & 1 \\ 4 & 5 & -2 \end{bmatrix}$.

1. Find A_{21}
2. Find A_{22}
3. Find A_{23}
4. Use the above information to calculate the following:

   $a_{31}A_{21} + a_{32}A_{22} + a_{33}A_{23} =$

   $a_{11}A_{21} + a_{12}A_{22} + a_{13}A_{23} =$

Example 3:

$$A(\text{adj } A) = \begin{bmatrix} 3 & -2 & 1 \\ 5 & 6 & 2 \\ 1 & 0 & -3 \end{bmatrix} \begin{bmatrix} -18 & -6 & -10 \\ 17 & -10 & -1 \\ -6 & -2 & 28 \end{bmatrix} =$$

$$(\text{adj } A)A = \begin{bmatrix} -18 & -6 & -10 \\ 17 & -10 & -1 \\ -6 & -2 & 28 \end{bmatrix} \begin{bmatrix} 3 & -2 & 1 \\ 5 & 6 & 2 \\ 1 & 0 & -3 \end{bmatrix} =$$

Example 4:

Consider the linear system $A\mathbf{x} = \mathbf{b}$ where $A = \begin{bmatrix} 1 & 2 & 0 \\ 0 & 2 & -3 \\ 0 & -2 & 0 \end{bmatrix}$ and $\mathbf{b} = \begin{bmatrix} 1 \\ 4 \\ -3 \end{bmatrix}$.

Find the unique solution $\mathbf{x}$ using Cramer's Rule.