Math 265
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Class Handout #3

Section 2.1 Echelon form of a matrix

Elementary Row Operations:

- Interchange rows ($r_i \leftrightarrow r_j$)
- Add a multiple of one row to another ($kr_i + r_j \to r_j$)
- Multiply a row by a nonzero scalar ($kr_i \to r_i$)

Definition: Two matrices A and B are row equivalent if there is a sequence of elementary row operations that transforms A into B.

Definition: A matrix A is in *reduced row echelon form* (ref) if it satisfies the following properties:

1. The first nonzero entry in a nonzero row is a 1, called a **leading one** of its row.
2. For each nonzero row, the leading one occurs to the right of the leading 1 in the row directly above it.
3. All zero rows are grouped at the bottom of the matrix.
4. If a column contains a leading one, then all other entries in that column are zero.

Definition: A matrix satisfying only properties 1, 2, and 3, above is said to be in *row echelon form* (ref).

Theorem 2.1: Every matrix A is row equivalent to a matrix in row echelon form.

Definition: The first column of a matrix A with a nonzero entry is called a **pivot column** and the first nonzero entry in a pivot column is called the **pivot**.

Theorem 2.2: Every matrix is row equivalent to a unique matrix in reduced row echelon form.

Section 2.2: Solving linear systems

Theorem 2.3: Suppose that $[A \mid \mathbf{b}]$ and $[C \mid \mathbf{d}]$ are row equivalent augmented matrices representing systems of linear equations. Then, the system $A\mathbf{x} = \mathbf{b}$ is equivalent to the system $C\mathbf{x} = \mathbf{d}$.

To solve a system of linear equations we use **Gaussian Elimination** to row reduce the matrix $[A \mid \mathbf{b}]$ to a matrix in row echelon form and then use back substitution, or we use **Gauss-Jordan Elimination** and row reduce $[A \mid \mathbf{b}]$ to a matrix in reduced row echelon form.

Note: If the row echelon form of an augmented matrix representing a system does not have a leading one in every column, always let the variables corresponding to columns without leading ones be the variables you solve in terms of (the ones you set to be free parameters).

Theorem 2.4: *(The more unknowns theorem)* A homogeneous system of m linear equations in n unknowns always has a nontrivial solution if $m < n$, that is, if the number of variables exceeds the number of equations.

Relationship between solutions to non homogeneous and homogeneous systems: Let $A\mathbf{x} = \mathbf{b}$, where $\mathbf{b} \neq \mathbf{0}$, be a consistent linear system. If $\mathbf{x}_p$ is a particular solution to $A\mathbf{x} = \mathbf{b}$ and $\mathbf{x}_h$ is a solution to the corresponding homogeneous system $A\mathbf{x} = \mathbf{0}$, then $\mathbf{x}_p + \mathbf{x}_h$ is also a solution to $A\mathbf{x} = \mathbf{b}$.