Math 265
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Class Handout #2

Recall from last class:

- Matrices are multiplied "row-by-column".

  Side note: Just because AB is defined does not mean that BA is defined. And even when AB and BA are defined, it is possible for $AB \neq BA$.

- Matrix-vector multiplication (row-by-column) and matrix-vector multiplication as a linear combination

  Side note: can think of the product matrix AB as "A times the columns of B"

- Linear systems can be represented by matrix-vector products $A\mathbf{x} = \mathbf{b}$

A is called the **coefficient matrix**, and adjoining the column $\mathbf{b}$ to A we get the augmented matrix $[A \mid \mathbf{b}]$ representing the linear system.

Very important: $A\mathbf{x} = \mathbf{b}$ is consistent if and only if the vector $\mathbf{b}$ is a linear combination of the columns of the coefficient matrix A.

Section 1.4: Algebraic Properties of Matrix Operations

Theorem 1.1 Properties of matrix addition: Let A, B, and C be $m \times n$ matrices.

1. $A + B = B + A$.
2. $A + (B + C) = (A + B) + C$.
3. There is a unique $m \times n$ matrix O, called the **zero matrix**, such that $A + O = A$.
4. For each $m \times n$ matrix A, there exists a unique $m \times n$ matrix D such that $A + D = O$. $D = -A$ is called the negative of A.

Theorem 1.3 Properties of scalar multiplication: If r and s are real numbers and A and B are matrices of the appropriate sizes, then

1. $r(sA) = (rs)A$.
2. $(r + s)A = rA + sA$.
3. $r(A + B) = rA + rB$.
4. $A(rB) = r(AB) = (rA)B$.

Theorem 1.2 Properties of matrix multiplication: If A, B, and C are matrices of the appropriate sizes, then

1. $A(BC) = (AB)C$.
2. $(A + B)C = AC + BC$.
3. $C(A + B) = CA + CA$.

Question: What property is missing from this list?

Two other peculiarities through example:

Example 1: Let $A = \begin{bmatrix} 1 & 2 \\ 2 & 4 \end{bmatrix}$ and let $B = \begin{bmatrix} 4 & -6 \\ -2 & 3 \end{bmatrix}$. What is AB?

Example 2: Let $A = \begin{bmatrix} 1 & 2 \\ 2 & 4 \end{bmatrix}$, $B = \begin{bmatrix} 2 & 1 \\ 3 & 2 \end{bmatrix}$, and $C = \begin{bmatrix} -2 & 7 \\ 5 & -1 \end{bmatrix}$. Calculate AB and AC.

Theorem 1.3 Properties of Transpose: If r is a scalar and A and B are matrices of the appropriate sizes, then

1. $(A^T)^T = A$.
2. $(A+B)^T = A^T + B^T$.
3. $(AB)^T = B^T A^T$.
4. $(rA)^T = rA^T$.

Section 1.5: Special Types of Matrices and Partitioned Matrices

What is special about the following matrices?

$$A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & -3 \end{bmatrix} \quad B = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{bmatrix} \quad C = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$D = \begin{bmatrix} 1 & 3 & 3 \\ 0 & 3 & -5 \\ 0 & 0 & 2 \end{bmatrix} \quad E = \begin{bmatrix} 1 & 0 & 0 \\ -2 & 2 & 0 \\ 3 & 5 & 3 \end{bmatrix}$$

$$F = \begin{bmatrix} 1 & -2 & 3 \\ -2 & 4 & 5 \\ 3 & 5 & 6 \end{bmatrix} \quad G = \begin{bmatrix} 0 & 2 & 3 \\ -2 & 0 & -4 \\ -3 & 4 & 0 \end{bmatrix}$$