Math 265
Professor Priyam Patel
3/3/16

Class Handout #12

Recall from last time:

1. Basis for a vector space V: A set of vectors that spans V and is linearly independent.
2. Note: Basis for a vector space is not unique.
3. How to verify a set of vectors is a basis for a vector space or subspace (verify two properties).
4. Method for finding a basis for Span $\{\mathbf{v_1}, \mathbf{v_2}, \ldots, \mathbf{v_k}\}$.

Exercise 1:

Consider the vector space P_3 and let $S = \{t^3 + t^2 - 2t + 1, t^2 + 1, t^3 - 2t, 2t^3 + 3t^2 - 4t + 3\}$. Find a basis for the subspace $W = \text{Span } S$.

Theorem 4.10: If $S = \{\mathbf{v}_1, \mathbf{v}_2, \ldots, \mathbf{v}_n\}$ is a basis for a vector space V and $T = \{\mathbf{w}_1, \mathbf{w}_2, \ldots, \mathbf{w}_r\}$ is a set of linearly independent vectors in V then T has at most n vectors in it, that is $r \leq n$.

Corollary 4.1: Every basis of a vector space V has the same number of vectors in it.

Dimension: The dimension of a vector space or a subspace is the number of vectors in any basis for that space, and is denoted by dim V.

Note: We already know the dimensions of our favorite vector spaces!

Corollary 4.4: If a vector space V has dimension n, then any set of more than n vectors in V must be linearly dependent.

Corollary 4.5: If a vector space V has dimension n, then any set of less than n vectors in V cannot span V.

Theorem 4.12: Let V be an n–dimensional vector space.

- If $S = \{\mathbf{v}_1, \mathbf{v}_2, \ldots, \mathbf{v}_n\}$ is a linearly independent set of n vectors in V, then S is a basis for V.
- If $S = \{\mathbf{v}_1, \mathbf{v}_2, \ldots, \mathbf{v}_n\}$ spans V, then S is a basis for V.

Exercise 2:

Consider the vector space $\mathbb{R}^3$. Find a basis for the subspace W of all vectors $\begin{bmatrix} a \\ b \\ c \end{bmatrix}$ where $a = 2b$. What is the dim W?

Exercise 3:

Consider the vector space M_{22}. Find a basis for the subspace W of all vectors (2 x 2 matrices) A such that tr $A = 0$. What is the dim W?

Exercise 4:

Consider the subspace W of P_2 formed by all polynomials $at^2 + bt + c$ where $a - b - c = 0$. Find a basis for W. What is dim W?

Recall: The *null space* of a matrix A, null A, is the solution space for the homogeneous system $A\mathbf{x} = \mathbf{0}$

Definition: The dimension of null A is called the *nullity* of A.

Exercise 5:

Let $A = \begin{bmatrix} 1 & -1 & 1 \\ 1 & -2 & 0 \\ 2 & -3 & 2 \end{bmatrix}$. Find a basis for null A and find the nullity of A.

Theorem: The spanning vectors in the solution set to the homogenous system $A\mathbf{x} = \mathbf{0}$ are linearly independent and therefore form a basis for null A.

Exercise 6:

Suppose A is a 3 x 5 matrix and we reduce the augmented matrix $\begin{bmatrix} A & \mathbf{0} \end{bmatrix}$ to $\begin{bmatrix} R & \mathbf{0} \end{bmatrix}$, where R is in reduced row echelon form and has 3 pivot positions. What is the nullity of A?

Suppose A is a 6 x 4 matrix and we reduce the augmented matrix $\begin{bmatrix} A & \mathbf{0} \end{bmatrix}$ to $\begin{bmatrix} R & \mathbf{0} \end{bmatrix}$, where R is in reduced row echelon form and has 2 pivot positions. What is the nullity of A?

Can you think of a general rule for computing the nullity of an m x n matrix A where the RREF R of A has r pivot positions?

Exercise 7:

Let $A = \begin{bmatrix} 1 & 1 & 4 & 1 & 2 \\ 0 & 1 & 2 & 1 & 1 \\ 0 & 0 & 0 & 1 & 2 \\ 1 & -1 & 0 & 0 & 2 \\ 2 & 1 & 6 & 0 & 1 \end{bmatrix}$. Find the nullity of A.