MATH 351 EXAM 3 REVIEW

1. Rough outline of what we have covered (Penney Sections 2.3, 3.1, 3.2, 3.3, 4.1, 4.2, 5.1, 5.2)

- Rank of A and the solution set to the system Ax=b – Theorem 5, Theorem 7, Definition 4, Theorem 8 and Theorem 9 Pages 144-146
- Nonsingular matrix
- Nonsingular matrix theorem – Theorem 9 and Proposition 2 Pages 146-147
- Linear transformations and linearity properties
- Matrix transformations
- Obtaining the matrix A for a linear transformation $T : \mathbb{R}^n \longrightarrow \mathbb{R}^m$
- Identity transformation and identity matrix
- Matrix multiplication, properties of matrix multiplication, and composition of transformations
- Inverses of matrices and inverse transformations
- Invertibility and nonsingularity– Theorem 2 Page 195
- Determinants – Compute the determinant of a matrix via cofactor expansion (in small cases and when there is an obvious row to expand along). Compute the determinant of an upper triangular or lower triangular matrix. Compute the determinant of a matrix by using row operations to put it in upper triangular form. Understand the effect that row operations have on the determinant of a matrix. Quickly determine whether a matrix is invertible using determinant.
- Eigenvalues and eigenvectors, characteristic polynomials, multiplicity of eigenvalues, eigenspaces and finding their bases, diagonalizability of a matrix of $\mathbb{R}$ (see the outline below for the procedure for diagonalization).

This list is not meant to be exhaustive. I would suggest reading through all of the sections of Penney we have covered, reviewing all definitions and making sure you understand why the Propositions and Theorems are true.

Procedure for diagonalizing matrices:

(1) Find the eigenvalues of A, that is find the roots of the characteristic polynomial of A. If there are no complex eigenvalues then proceed to step 2. If there are any complex eigenvalues then A is not diagonalizable.
(2) Find the dimensions of the corresponding eigenspaces by finding dim Null$(A - \lambda_0 I_n)$ = nullity $(A - \lambda_0 I_n)$. If nullity $(A - \lambda_0 I_n)$ is equal to the multiplicity of λ_0 for each eigenvalue λ_0 then A is diagonalizable.
(3) To diagonalize A, start by finding a basis of eigenvectors for the eigenspace corresponding to each eigenvalue. That is, find a basis for Null$(A - \lambda_0 I_n)$.
(4) Form the invertible matrix Q whose column vectors are the eigenvectors you found in step 3. Form the diagonal matrix D whose entries along the main diagonal are the corresponding eigenvalues (the order of the values is determined by how you put down the eigenvectors as the columns of Q).

2. Practice Questions from Textbook

Please take the time to practice these textbook problems. I think the problems for the sections we have covered for this exam are very good.

Section 2.3 TF: 4, 5, 6, 8 Exs: 2, 3, 5, 12, 21, 22
Section 3.1 TF: 1, 6 Exs: 14, 15, 20, 27
Section 3.2 TF: 5, 7 Exs: 1, 3, 26, 29
Section 3.3 TF: 6, 8 Exs: 3, 10, 18, 20, 21, 22, 22, 23
Section 4.1 TF: 1-5 Exs: 1, 9
Section 4.2 TF: 1, 4 Exs: 1, 5, 15, 16
Section 5.1 TF: 1, 2, 5, 6, 7, 8, 9, 11 Exs: 1, 6, 9, 15
Section 5.2 TF: 1, 2 Exs: 2, 4, 5, 12

3. Extra Problems

(1) For matrices A, B, if AB is defined must BA also be defined?
(2) Is the matrix product AA^T always defined?
(3) Is every invertible matrix square? Is every square matrix invertible?
(4) What can you say about the reduced row echelon form of an invertible matrix?

(5) Suppose that

$$A = \begin{bmatrix} 1 & 3 \\ -1 & 2 \end{bmatrix}, \quad B = \begin{bmatrix} 3 & 2 & -1 \\ -1 & 2 & 0 \end{bmatrix}, \quad C = \begin{bmatrix} -2 & 3 \\ 3 & -3 \\ 4 & 1 \end{bmatrix} \text{ and } D = \begin{bmatrix} 0 & 6 & 2 \\ -3 & 0 & 4 \end{bmatrix}.$$

Which of the following are defined? Calculate those that are:
(a) AB (b) AD^T (c) BAC (d) CAB (e) C^TC

(6) What is the relationship between the determinant of a matrix and the determinant of its inverse?

(7) What is the relationship between the determinant of a matrix and the determinant of its transpose?

(8) Is $\det(AB) = (\det A)(\det B)$? Is $\det(A+B) = (\det A) + (\det B)$?

(9) If A is a 2×2 matrix, what is $\det(-A)$? What if A is 3×3? If A is a 2×2 matrix, what is $\det 2A$? What if A is 3×3?

(10) If A is an 4 x 4 matrix where rank $A = 4$, what can you say about det A?

(11) If A is a 4 x 4 matrix and $\det A = -3$ what is rank A?

(12) If A is a 3 x 3 matrix and $\det A = 0$, can you determine whether the column vectors of A form a basis for $\mathbb{R}^3$? What are the possible values for rank A?

(13) Which of the following are subspaces of M_{22}?
  (a) The set of 2 x 2 matrices with zero determinant.
  (b) The set of 2 x 2 nonsingular matrices.

Classify each of the following statements as TRUE or FALSE:

(1) $(AB)^T = B^TA^T$.
(2) $(A+B)^T = A^T + B^T$.
(3) AA^T is always symmetric.
(4) AA^T is always nonsingular.
(5) AA^T always equals A^TA.
(6) A square matrix A is nonsingular if and only if $\det A = 0$.
(7) If A is an n x n matrix with n distinct real eigenvalues, then A is diagonalizable.
(8) If A is a square matrix with an eigenvalue λ_0 of multiplicity k, then the dimension of the eigenspace corresponding to λ_0 is also k.
(9) If A is an $n \times n$ matrix and $\lambda = 0$ is an eigenvalue of A, then A is not invertible.
(10) If A is a 3×3 matrix with eigenvalues $\lambda = 1, -5, 4$, then $\det A = -20$.