# MATH 351 EXAM 1 REVIEW

## 1. Outline of what we have covered– roughly Sections 1.1, 1.2, 1.3 and some of 1.4 of Penney's Linear Algebra

- Matrices: size of matrix, adding matrices (when this is defined), scalar multiple of matrix, transpose of a matrix
- $\mathbb{R}^n$ as the set of all $n$-vectors
- Linear combinations, linear dependence and independence
- Span and geometric concept of what the span of one or two vectors in $\mathbb{R}^n$ looks like
- Vector Space Properties and the vector spaces $\mathbb{R}^n$, $M(m, n)$, $\mathcal{F}(\mathbb{R})$, $\mathcal{P}$ and $\mathcal{P}_n$
- Systems of linear equations: equivalent systems, quickly determining whether a proposed solution to a linear system is in fact a solution, types of solutions (one, none, inf many), parametric form of the solution set and the geometric interpretation of the solution set using parametric form, consistent and inconsistent systems, homogeneous systems, rank of a system, augmented and coefficient matrix for a system
- Gaussian Elimination: elementary row operations, row equivalent matrices, EF, REF, pivot variables, free variables, the "More Unknowns Theorem"
- Checking whether vector is in span of some others when working in the vector spaces $\mathbb{R}^n$, $\mathcal{P}_n$, $M(m, n)$
- Checking whether a set of vectors spans the whole vector space $\mathbb{R}^n$, $\mathcal{P}_n$ or $M(m, n)$
- Definition of the column space of a matrix and checking whether a given vector is in the column space for a particular matrix.
- A linear system is consistent if and only if the vector of constants $\mathbf{b}$ is in Col $A$, where $A$ is the coefficient matrix.
- Matrix vector multiplication and the linearity properties.

This is a quick outline, I may have missed one or two things. Anything and everything we have discussed in class could show up on the exam!

## 2. Practice Quetions from Textbook

- Penney Section 1.1 TF #1 - 10, Exercises #3, 4, 5 , 10, 12, 13, 14, 16, 17, 20, 24, 26, 27, 31, 32, 34a, d,e,h, 35, 36
- Penney Section 1.2 TF #1, 2, 3, 4, 5, 6, Exercises #1-8
- Penney Section 1.3 TF #1, 2, 4, Exercises #1, 2, 3a, b, c, d, 5, 7, 8, 20, 21, 22

- Penney Section 1.4 Exercises # 1 (For more practice for this section see some of the additional problems below)

## 3. Some extra questions

- Suppose that $A = \begin{bmatrix} 1 & 3 \\ -1 & 2 \end{bmatrix}$, $B = \begin{bmatrix} 3 & 2 & -1 \\ -1 & 2 & 0 \end{bmatrix}$, $C = \begin{bmatrix} 2 & 3 \\ 3 & -3 \\ 4 & 1 \end{bmatrix}$, $D = \begin{bmatrix} 0 & 6 & 2 \\ -3 & 0 & 4 \end{bmatrix}$.

  Which of the following quantities are defined? Calculate those that are.

  a. $-3D$
  b. $B + 2C$
  c. $A + B$
  d. $B^T + 2C$
- Is there a $2 \times 2$ matrix $A$ such that the homogeneous system represented by $[A\ \mathbf{0}]$ has only $\mathbf{0}$ as a solution?
- Is there a $2 \times 2$ matrix $A$ such that the homogeneous system represented by $[A\ \mathbf{0}]$ has infinitely many solutions?
- Is there a $2 \times 2$ matrix $A$ such that the homogeneous system represented by $[A\ \mathbf{0}]$ has no solutions?
- For each of the following, give an example of a matrix satisfying the criteria, or explain why no such matrix exists:
  (1) a 3 x 4 matrix $R$ in RREF such that for *every* $\mathbf{c} \in \mathbb{R}^3$ the augmented matrix $[R\ \mathbf{c}]$ represents a consistent system.
  (2) a 3 x 4 matrix $R$ in RREF such that the the augmented matrix $[R\ \mathbf{0}]$ represents a system that has a *unique* solution.
  (3) a 4 x 3 matrix $R$ in RREF such that for *every* $\mathbf{c} \in \mathbb{R}^4$ the augmented matrix $[R\ \mathbf{c}]$ represents a system that has at least one solution.
  (4) a 4 x 3 matrix $R$ in RREF such that the the augmented matrix $[R\ \mathbf{0}]$ represents a system that has a *unique* solution $\mathbf{x} \in \mathbb{R}^3$
  (5) a 4 x 4 matrix $R$ in RREF such that the the augmented matrix $[R\ \mathbf{0}]$ represents a system that has a *nontrivial* solution $\mathbf{x} \in \mathbb{R}^4$
  (6) Let $A$ be an $m \times n$ matrix. What is the definition of the column space, Col$(A)$, of $A$? For what value of $k$ is Col $A$ a subspace of $\mathbb{R}^k$?
  (7) Is it true that for any vector $\mathbf{b} \in$ Col$(A)$ the system represented by the augmented matrix $[A\ \mathbf{b}]$ is consistent? Please explain.
  (8) If $A$ is an $m \times n$ matrix and Col$(A) = \mathbf{R}^m$, is the system $A\mathbf{x} = \mathbf{b}$ always consistent for any $\mathbf{b} \in \mathbf{R}^m$?

(9) Is $\mathbf{b} = \begin{bmatrix} 14 \\ 0 \\ -3 \end{bmatrix}$ in the column space of $A = \begin{bmatrix} 3 & 1 & 1 \\ -1 & 1 & 4 \\ 2 & 1 & -5 \end{bmatrix}$?

(10) Suppose that $\mathbf{u}_1 = \begin{bmatrix} 1 \\ 3 \\ -2 \end{bmatrix}$, $\mathbf{u}_1 = \begin{bmatrix} 3 \\ 5 \\ -3 \end{bmatrix}$ and $\mathbf{u}_3 = \begin{bmatrix} 1 \\ -1 \\ 2 \end{bmatrix}$. Is the span of the set of vectors $S = \{\mathbf{u}_1, \mathbf{u}_2, \mathbf{u}_3\}$ all of $\mathbb{R}^3$? Justify your answer.

(11) Is $A = \begin{bmatrix} 5 & 1 \\ -1 & 9 \end{bmatrix}$ in span $\left\{ \begin{bmatrix} 1 & -1 \\ 0 & 3 \end{bmatrix}, \begin{bmatrix} 1 & 1 \\ 0 & 2 \end{bmatrix}, \begin{bmatrix} 2 & 2 \\ -1 & 1 \end{bmatrix} \right\}$?

(12) In $\mathcal{P}_2$, let $\mathbf{p_1} = 3x^2 - x + 2$, $\mathbf{p_2} = x^2 + x + 1$, and $\mathbf{p_3} = x^2 + 4x - 5$. Write $\mathbf{q} = 14x^2 - 3$ as a linear combination of $\{\mathbf{p_1}, \mathbf{p_2}, \mathbf{p_3}\}$.

(13) Does the set of vectors $\{\mathbf{p_1}, \mathbf{p_2}, \mathbf{p_3}\}$ from Exercise 5 span all of $\mathcal{P}_2$?

(14) Does the set $\left\{ \begin{bmatrix} 1 & 0 \\ 1 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 1 \\ 2 & 2 \end{bmatrix}, \begin{bmatrix} 1 & 1 \\ 2 & 2 \end{bmatrix}, \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix} \right\}$ span $M(2,2)$?

(15) Is the matrix $\begin{bmatrix} 3 & -2 \\ -4 & -4 \end{bmatrix}$ in span $\left\{ \begin{bmatrix} 1 & 0 \\ 1 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 1 \\ 2 & 2 \end{bmatrix}, \begin{bmatrix} 1 & 1 \\ 2 & 2 \end{bmatrix} \right\}$?

(16) If $A$ is a $2 \times 3$ matrix such that $A \begin{bmatrix} -1 \\ 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$ and $A \begin{bmatrix} 0 \\ 2 \\ 1 \end{bmatrix} = \begin{bmatrix} -1 \\ 3 \end{bmatrix}$, then what is $A \begin{bmatrix} -5 \\ 1 \\ -2 \end{bmatrix}$? *(Hint: Use the linearity properties of matrix vector products.)*